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Chiral-symmetry restoration in clustered hadronic matter

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Abstract. Chiral-symmetry restoration is usually discussed in the context of quark matter, a system of deconfined quarks. However, many systems like stable nuclei and neutron stars have quarks confined within nucleons. In the present paper we use a Fermi sea of three-quark clusters instead of a Fermi sea of deconfined quarks to investigate the in-medium quark condensate. We find that an enhancement of the chiral breaking in clustered matter as claimed in the literature is not a consequence of the clustering but rather dependent on the microscopic model dynamics.

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1 Introduction

One of the key issues of contemporary nuclear physics is to understand how hadronic properties change in a hadronic medium at finite density and temperature. In general, low-energy hadronic properties, such as meson and baryon masses, are determined by the chiral condensate. In ultra-relativistic heavy-ion collisions it is possible to produce matter at temperatures that, much likely, lead to color deconfinement and to the chiral phase transition. The chiral phase transition is also related to the behavior of the condensate as a function of the temperature. Therefore, both at low- and high-energy nuclear physics, fundamental questions on the physics of strong interactions are related to the in-medium behavior of the quark condensate.

Chiral-symmetry restoration at finite baryon density is usually discussed in the context of quark matter, *i.e.* a system of deconfined quarks. For many systems like stable nuclei and neutron stars, quarks are confined within nucleons and are far from being free. The usual scenario for in-medium chiral restoration is Pauli blocking. That is, the quarks that occupy uniformly the Fermi sea contribute positively to the quark condensate and when this positive contribution is added to the negative contribution coming from the quarks in the Dirac sea, the absolute value of the condensate decreases. This decrease is the signal of the chiral restoration. The original model of Nambu and Jona-Lasinio [1] (NJL), reinterpreted in terms of quarks, has been the prototype model for such investigations [2,3].

In a recent publication [4] it was argued that once quarks are confined within nucleons, there is the possibility of an in-medium enhancement of the chiral breaking. This is because the momentum distribution of confined quarks is very different from that of free quarks and, depending on the dynamics of the constituent quarks, the Pauli blocking becomes less effective than in a Fermi sea of quarks. This is a very interesting result, with far reaching consequences for many experiments planned for the near future.

The author of ref. [4] substantiated his claim on the basis of a model inspired on Coulomb gauge QCD [5] (CgQCD). This model is basically a generalization of the NJL model in that instead of the point-like NJL interaction, it uses a confining potential. The wave functions of the clusters of three quarks were obtained using a variational approach [6]. In the present paper we reinvestigate this issue using the same ideas of ref. [4], but use a slightly different philosophy with respect to the dynamics. We employ the NJL model to generate the massive constituent quarks, and then confine them with a nonrelativistic potential in a way to obtain a nucleon cluster of three constituent quarks. Then we construct a Fermi sea of nucleon clusters and obtain the gap equation. In this way, one is able to investigate the differences in the Pauli blocking within the same basic underlying NJL model.

2 The model

We start considering chiral-symmetry restoration in quark matter in the context of the NJL model. For our purposes in the present paper it is sufficient to use the following two-flavor Lagrangian density [2,3]:

$$\mathcal{L} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi + G_S\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\boldsymbol{\tau}\gamma_5\psi)^2\right], \qquad (1)$$

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where ψ is the quark field operator (with color and flavor indices suppressed). For simplicity we have taken zero current quark masses. The energy density of quark matter,

$$\mathcal{E}_{\rm QM} = \frac{1}{V}\int {\rm d}^3x\, \langle QM|T^{00}(x)|QM\rangle, \tag{2}$$

where $|QM\rangle$ is the many-quark state and T^{00} is the zero-zero component of the energy-momentum stress-tensor

$$T^{\mu\nu}(x) = i\bar{\psi}\gamma^\mu\partial^\nu\psi(x) - g^{\mu\nu}\mathcal{L}. \tag{3}$$

In the mean-field approximation, the state $|QM\rangle$ can be written in a schematic way as

$$|QM\rangle = q^\dagger(\boldsymbol{k}_{\rm F})\cdots q^\dagger(\boldsymbol{k}_2)q^\dagger(\boldsymbol{k}_1)|\Omega\rangle, \tag{4}$$

where $q^\dagger(\boldsymbol{k})$ are constituent quark creation operators and $|\Omega\rangle$ is the vacuum of constituent quarks —we have suppressed color-flavor-spin indices. In this approximation, and not using the equations of motion of the quark field operator, $\mathcal{E}$ can be written as

$$\mathcal{E}_{\rm QM} = -(2N_fN_c)\int\frac{{\rm d}^3k}{(2\pi)^3}\frac{k^2}{E^*(k)}[1-n(k)] -G_S\,(2N_fN_c)^2\left\{\int\frac{{\rm d}^3k}{(2\pi)^3}\frac{M^*}{E^*(k)}[1-n(k)]\right\}^2, \tag{5}$$

where N_f and N_c are respectively the number of quark flavors and colors, $E^*(k) = (k^2+M^{*\,2})^{1/2}$, M^* is the in-medium constituent quark mass, and $n(k)$ is the quark density. For the state given in eq. (4), the quark density is simply

$$n(k) = \theta(k_{\rm F}-k), \tag{6}$$

with $k_{\rm F}$ being the quark Fermi momentum. The normalization of quark density is such that

$$\int_0^\infty {\rm d}^3k\, n(\boldsymbol{k}) = \frac{4\pi}{3}k_{\rm F}^3 \equiv \frac{\rho_q}{\gamma_q}, \tag{7}$$

where ρ_q and γ_q are respectively the quark density and degeneracy. The integrals in eq. (2) are ultraviolate divergent and a cutoff Λ has to be introduced. Since the model is nonrenormalizable, Λ is an additional parameter of the model.

The in-medium constituent quark mass M^* can be obtained by minimizing $\mathcal{E}_{\rm QM}$ with respect to M^*. The minimization leads to the usual gap equation [2,3]

$$M^* = 2\,G_S\,(2N_fN_c)\int^\Lambda\frac{{\rm d}^3k}{(2\pi)^3}\frac{M^*}{E^*(k)}[1-n(k)]. \tag{8}$$

Given the solution of this equation, one can calculate the in-medium quark condensate. The condensate of quark flavor $q(=u,d)$ is given by

$$\langle\bar{q}q\rangle = -2N_c\int^\Lambda\frac{{\rm d}^3k}{(2\pi)^3}\frac{M^*}{E^*(k)}[1-n(k)] = 2N_c\int^\Lambda\frac{{\rm d}^3k}{(2\pi)^3}\frac{M^*}{E^*_-(k)} + 2N_c\int_0^{k_{\rm F}}\frac{{\rm d}^3k}{(2\pi)^3}\frac{M^*}{E^*_+(k)}, \tag{9}$$

where $E^*_\pm(k) = \pm(k^2+M^{*\,2})^{1/2}$.

The last two lines in eq. (9) are written in a way to make clear the physical interpretation of the effect of the Fermi sea of constituent quarks: the positive-energy quarks of the Fermi sea add a positive contribution to the condensate and therefore cancel the contributions of the negative-energy quarks of the Dirac sea. The positive contribution of the Fermi sea is a monotonically increasing function of quark Fermi momentum $k_{\rm F}$, and leads to a partial restoration of chiral symmetry in a medium with a finite quark density. This is the basis of the common wisdom of the in-medium decrease of (the absolute value of) the condensate.

Next, we populate the positive-energy states with clusters of three constituent quarks confined by a potential. The Fermi sea is now is written as

$$|NM\rangle = N^\dagger(\boldsymbol{p}_{\rm F})\cdots N^\dagger(\boldsymbol{p}_2)N^\dagger(\boldsymbol{p}_1)|\Omega\rangle, \tag{10}$$

where the $N^\dagger(\boldsymbol{p})$ are nucleon creation operators. These are written in terms of the constituent quark creation operators as

$$N^\dagger(\boldsymbol{p}) = \int {\rm d}^3k_1{\rm d}^3k_2{\rm d}^3k_3\,\Psi_{\boldsymbol{p}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) \times q^\dagger(\boldsymbol{k}_1)q^\dagger(\boldsymbol{k}_2)q^\dagger(\boldsymbol{k}_3), \tag{11}$$

where $\Psi_{\boldsymbol{p}}$ is the Fock-space amplitude for a nucleon with c.m. momentum $\boldsymbol{p}$. In the equations above we have suppressed nucleon spin-isospin indices and, as usual, quark color-spin-flavor indices.

In order to confine the constituent quarks, we add a nonrelativistic two-body harmonic-oscillator potential and write for $\Psi_{\boldsymbol{p}}$

$$\Psi_{\boldsymbol{p}}(\boldsymbol{k}_1\boldsymbol{k}_2\boldsymbol{k}_3) = \delta(\boldsymbol{p}-\boldsymbol{k}_1-\boldsymbol{k}_2-\boldsymbol{k}_3)\Phi_{\boldsymbol{p}}(\boldsymbol{k}_1\boldsymbol{k}_2\boldsymbol{k}_3), \tag{12}$$

and for Φ we make a Gaussian ansatz [6],

$$\Phi_{\boldsymbol{p}}(\boldsymbol{k}_1\boldsymbol{k}_2\boldsymbol{k}_3) = \left(\frac{3b^4}{\pi^2}\right)^{\frac{3}{4}}\exp\left[-\frac{b^2}{2}\sum_{i=1}^3\left(\boldsymbol{k}_i-\frac{\boldsymbol{p}}{3}\right)^2\right], \tag{13}$$

where b is the variational parameter. This leads to the following normalization for the amplitude Ψ:

$$\langle\Psi_{\boldsymbol{p}'}|\Psi_{\boldsymbol{p}}\rangle = \delta(\boldsymbol{p}'-\boldsymbol{p}). \tag{14}$$

In the mean-field approximation, and neglecting effects of quark-exchange between different nucleons [7], the energy density of nuclear matter,

$$\mathcal{E}_{NM} = \frac{1}{V}\int {\rm d}^3x\,\langle NM|T^{00}(x)|NM\rangle, \tag{15}$$

is given by precisely the same form as in eq. (5), but with a different quark momentum distribution $n(k)$. For the ansatz in eqs. (12) and (13), the quark momentum distribution in the state of clustered matter, eq. (10), is given by

$$n(\boldsymbol{k}) = \int_0^{p_{\rm F}}{\rm d}^3p\,n_{\boldsymbol{p}}(\boldsymbol{k}), \tag{16}$$

with

$$n_{\boldsymbol{p}}(\boldsymbol{k}) = \left(\frac{3b^2}{2\pi}\right)^{\frac{3}{2}} \exp\left[-\frac{3}{2}b^2\left(\boldsymbol{k} - \frac{1}{3}\boldsymbol{p}\right)\right]. \qquad (17)$$

The quark distribution function is normalized as

$$\int_0^\infty \mathrm{d}^3k\, n(\boldsymbol{k}) = \frac{4\pi}{3}p_\mathrm{F}^3 \equiv \frac{\rho_N}{\gamma_N}. \qquad (18)$$

For symmetrical nuclear matter,

$$\frac{\rho_N}{\gamma_N} = \frac{\rho_q}{\gamma_q}, \qquad (19)$$

and therefore quarks and nucleons have the same Fermi momentum, $k_\mathrm{F} = p_\mathrm{F}$.

The gap equation has the same form as in eq. (8), but now it is coupled with the equation for b, the variational parameter of the nucleon amplitude. Numerical results are presented in the next section.

3 Numerical results

Initially we fix the parameters to vacuum properties. Using $\Lambda = 0.7$ GeV and $G_S\Lambda^2 = 2.14$, we obtain for the constituent quark mass the value $M = 335$ MeV and for the condensate $\langle\bar{q}q\rangle = (-268\ \mathrm{MeV})^3$. We adjust the spring constant of the confining potential to obtain for the size parameter of the nucleon the value $b = 0.5$ fm. Then the nucleon mass can be fitted by adding a negative constant to the potential, as usually done in quark potential models.

Next, we solve the gap equation and minimize the nucleon mass with respect to the size parameter. The solutions will be M^* and b^*. We are interested in the behavior of the quark condensate as a function of the nucleon density. In fig. 1 we present the ratio of the in-medium to vacuum condensates, as a function of the ratio of the nucleon density to the saturation density of normal nuclear matter. The figure shows that the (absolute value of the) condensate in medium decreases monotonically with the density. In addition, clearly there is no sign of reversing this tendency.

The result shown in fig. 1 is easy to understand. The gap equation always gives a density dependence of the mass for the constituent quarks that is monotonically decreasing as the density increases. Therefore, the kinetic energy of the quarks in the nucleon bound state tends to increase, leading to the swelling of the nucleons. In order to obtain an increase of $|\langle\bar{q}q\rangle|$ in medium, the nucleon should shrink, as argued in ref. [4]. In the CgQCD model of ref. [4], there is a spin-spin force that induces a very strong attraction as the symmetry becomes restored. This attraction then leads to a shrinking of the nucleon. In the present model this is impossible, and the usual scenario of chiral symmetry restoration is maintained.

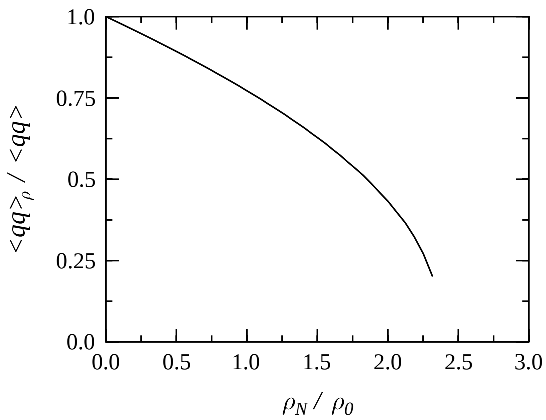

Fig. 1. The ratio of the in-medium to vacuum condensates as a function of the ratio of the nucleon density to the saturation density of normal nuclear matter.

4 Conclusions and perspectives

In the present paper we have shown that the question of an enhancement of the chiral breaking in hadronic matter as argued in ref. [4] seems to be model dependent. This is because the basic mechanism that could drive an enhancement depends crucially on a very attractive interaction that becomes stronger as the symmetry is restored. In ref. [4] the attraction comes from a spin-spin force that is a consequence of the very peculiar form of the confining potential used there. It would be interesting to investigate this question further with another spin-dependent interaction, such as transverse-gluon interactions that have been shown [5] to play an important role in the mechanism of dynamical chiral symmetry breaking.

Certainly the issue is very interesting and, in order to make progress, it is necessary to have a better understanding on the dynamics of the interaction that drives the symmetry breaking.

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